Preface

'Peristalsis' is a mechanism of pumping viscous fluids in ducts against an adverse pressure gradient by means of a series of moving contractile rings on the wall It is an inherent property of many of the smooth muscle tubes such as the gastrointestinal tract, bile duct, ureter and other glandular ducts wherein, the irritation at the epithelial lining of the duct can cause a contractile ring to spread along the tube, pushing the fluid contents ahead (Guyton, 1986) The fluids present in the ducts of a living body are called biofluids The major industrial application of this principle is in designing the roller pumps which are useful in pumping fluids without being contaminated due to the contact with the pumping machinery Even though peristalsis is a well known pumping phenomenon observed in biological systems for many decades, the first attempt to study the fluid mechanics of peristaltic transport is by Latham (1966) Following this experimental work, Burns & Parkes (1967) have developed a mathematical model for homogeneous fluids in a channel idealized under the assumptions of inertia free motion, due to an infinite train of peristaltic waves with small amplitudes The corresponding axisymmetric case has been discussed by Barton & Raynor (1968) Asymptotic solutions for the axisymmetric peristaltic motion in terms of the ratio of the small amplitude to the mean tube radius was given by Yin & Fung (1969) for a finite range of Reynolds number and wavelength The small amplitude assumption has been relaxed under infinite wavelength approximation by Shapiro *et al* (1969) For a complete review of work in the field, one may refer to Jaffrin & Shapiro (1971) and Rath (1980) The experimental verification of these models by Weinberg, Eckstein & Shapiro (1971) confirmed that the Lagrangian approach should be used to obtain the criterion for the backward leakage of fluid particles

The particulate flows under peristalsis started as a matter of interest with the work of Hung & Brown (1976) They have studied the nonlinear peristaltic flow in which a particle is pumped with the fluid Kaimal (1978) studied theoretically the peristaltic flow with uniformly distributed suspended particles under low Reynolds number and long-wavelength assumptions A pertubation solution for peristaltic transport of a fluid-particle mixture for small amplitudes and arbitrary Reynolds number and wave number was given by Srivastava & Srivastava (1989) Recently, Srivastava & Srivastava (1995) have studied the effects of Poiseuille flow on peristaltic transport of a particulate suspension

Another interesting branch of the peristaltic flow problems is the study of the effects of wave forms and tube geometries on the flow characteristics Mahrenholtz, Mank & Zimmermann (1978) have shown that the wave form strongly influences the pressure recordings when the Reynolds number is not negligible The peristaltic motion due to the propagation of lateral bending waves along the channel walls has been investigated by Wilson & Panton (1979) Rath (1982) has studied the peristaltic flow through a lobe-shaped tube under zero Reynolds number and long-wavelength assumptions A thesis on the peristaltic transport in a channel with flexible porous walls contained within channel with fixed walls has been presented by Reese (1988) and it contains the review of literature upto 1988

Most of the tubular organs in the living body contain a coating of a thick mucus secretion at the inner surface of the walls, which serves as a lubricant and an excellent protectant of the walls Its composition and hence the fluid properties differ in different ducts (Best & Taylor, 1958) Hence, the analysis of the peristaltic motion in the presence of a peripheral layer formed with a different fluid from that of the fluid to be transported is an integral part of the study of physiological motion in many ducts The major part of this thesis is devoted to the study of peristaltic transport of a biofluid in a tube with an inward coating by a different fluid forming a peripheral layer The relevant references for the two-fluid system are not given here in order to avoid repetition and they are cited appropriately in various chapters of the thesis

The assumptions in the work are that (i) the two fluids in contact are immiscible and incompressible, (ii) the peristaltic wave train is periodic and propagates with constant speed c, (iii) the pressure difference across the length of the tube is a constant, (iv) the tube length is an integral multiple of the wavelength and (v) the fluid interface is periodic with period same as that of the wall Under these assumptions, the flow becomes steady in the wave frame moving with constant speed c The peristaltic motion works as a pump when there is a pressure rise across the tube length It is also observed in nature, for e g in the intestines, that the peristaltic motion assists the flow driven by a pressure drop across the tube length, which is called copumping The commonly studied fluid characteristics of peristaltic pumping are (a) the quantity of fluid transported in one period of the wave, namely the time-averaged flux $\bar{Q}$ and its variation with the pressure rise or drop across one wavelength, (b) the mechanical efficiency, which is the ratio of the average rate of work done by the moving fluid against a pressure rise to the average rate of work done by the moving wall, (c) the trapping of a few fluid particles by the wave, which undergo a circulatory motion and are translated by the wave with the speed of the wave and (d) the reflux or backward leakage of fluid particles near the wall resulting from the incomplete occlusion

This thesis contains five chapters and it should be noted that the nomenclatures of the chapters are independent of each other

In the first chapter, the peristaltic pumping and copumping of two fluids in a circular tube are studied under long wavelength and low Reynolds number assumptions The fluid interface is obtained as a polynomial equation of sixth degree in core thickness Nonunique solutions for this equation are obtained in the flow region for a certain range of values of $\bar{Q}$ The effect of peripheral layer viscosity on $\bar{Q}$ and the mechanical efficiency are analysed The formation and growth of the trapping zone in the core for small values of $\bar{Q}$ and its shifting to the peripheral layer for sufficiently large values of $\bar{Q}$ are noticed It is observed that the trapped bolus volume in the peripheral layer increases with an increase in μ, which is the ratio of the viscosity of the peripheral layer fluid to that of the core The limits on $\bar{Q}$ for the appearance of the trapping zone in the core are obtained The trapping zone formed in the peripheral layer decreases in size with an increase in $\bar{Q}$ but it never disappears The development of complete trapping of the core fluid by the peripheral layer fluid with an increase in $\bar{Q}$ is demonstrated The effect of peripheral layer viscosity on the reflux layer is investigated It is also observed that the reflux occurs in the entire pumping range for all μ and it is absent in the entire range of copumping

The effects of curvature and inertia on the peristaltic transport of two fluids in a two dimensional channel are presented in Chapter 2 An asymptotic solution for the low Reynolds number flow is presented in powers of a geometric parameter which is the ratio of the channel width to the wavelength Velocity and stress balance at the interface at different orders are reduced and transferred to the known zeroth order interface An expression for the jump in the first order pressure across the interface is obtained through the balance of the normal stress and the solutions are presented upto first order For some non-zero wall curvature, the trapping in the core splits into three eddies, a larger bolus with two small boluses on either side, for $\mu > 1$ The trapped bolus volume decreases with an increase in curvature for all μ The inertia force mainly causes asymmetry in the streamline structure as pointed out by the earlier authors However, the trapped bolus being pushed forward for $\mu < 1$ is a new phenomenon Another interesting feature for a single fluid in copumping range is the displacement of the trapped bolus near the wall to the downstream side of the wave

Most of the ducts in the living body do not have perfectly circular cross sections and the evidence for them to be elliptic can be found in the literature Chapter 3 is devoted to the study of peristaltic transport in a pipe of elliptic cross section under long wavelength and low Reynolds number assumptions An appropriate elliptic cylindrical coordinate system is chosen to carry out the analysis The fluid interface is obtained in terms of a trancendental equation which, in the limit of circular geometry, reduces to the sixth degree polynomial in core thickness Nonunique solutions of this equation are also obtained within the flow regime for a certain range of values of $\bar{Q}$ The effect of peripheral layer viscosity on $\bar{Q}$ for a given pressure rise is studied for different eccentricities of the tube while the surface area and the bolus volume under the wave motion remain the same The peristaltic pump works against a larger pressure rise when the eccentricity is increased, for all values of the core and peripheral layer fluid viscosities The single fluid peristaltic flow in an elliptic pipe is studied as a particular case The pressure- flow rate characteristics and the mechanical efficiency are studied for different eccentricities Further, the volume of trapped bolus is observed to decrease with an increase in the eccentricity

Chapter 4 contains the study of two-layered peristaltic transport of power law fluids in an axisymmetric geometry under long wavelength and zero Reynolds number assumptions The model considered facilitates various combinations of choosing the core and the peripheral layer fluids to be shear thinning, shear thickening and Newtonian The interface between the two layers is determined from a transcendental equation in the core thickness The variation of the time mean flow $\bar{Q}$ with the pressure rise or drop over one wavelength Δp is studied It is observed that a negative mean flow is achieved under free pumping ($\Delta p = 0$) for atleast one wave form if one of the peripheral layer and core fluids is non-Newtonian The rheology of the peripheral layer fluid is a dominant factor in producing a negative or positive mean flow It is noticed that a sinusoidal wave always yields a positive mean flow for power-law fluids The trapped bolus volume for sinusoidal peristaltic wave is observed to decrease with an increase in the rate of shear thinning of the core and the peripheral layer fluids

Another intersetting aspect of the study of peristaltic transport is the pressure signature inside the bolus The manometric catheters used to measure the pressure in the bolus may also affect the pressure measurements, which is the motivation for the analysis presented in Chapter 5 The influence of an eccentrically inserted catheter on the peristaltic pumping in a tube is investigated under long wavelength, low

Reynolds number assumptions The radially asymmetric deformation of the wall arising through the eccentrically inserted catheter is taken into consideration by choosing an appropriate bipolar coordinate system The effect of the position and size of the catheter on pumping characteristics is studied The size of the catheter, when placed eccentrically, alters the pressure signature significantly inside the bolus, unlike the concentric case Further, the maximum pressure rise in one period of the peristaltic wave is observed to decrease with an increase in the eccentricity

The following papers are written based on the material embodied in this thesis

1 Peristaltic transport of two immiscible fluids in a circular tube (with Prof A Ramachandra Rao), J Fluid Mech , 298, pp 271-285 (1995)

2 Peristaltic pumping in a circular tube in the presence of an eccentric catheter (with Prof A Ramachandra Rao), J Biomech Engng , 117, pp - - (1995) pp 448-454

3 Peristaltic transport of a biofluid in a pipe of elliptic cross section (with Prof A Ramachandra Rao), J Biomech , 28 pp 45-52 (1995)

4 Effects of curvature and inertia on the peristaltic transport in a two-fluid system (with Prof A Ramachandra Rao), communicated to J Fluid Mech

5 Peristaltic transport of two-layered power fluids (with Prof A Ramachandra Rao), communicated to J Biomech Engng

6 Peristaltic transport of biofluids (with Prof A Ramachandra Rao), Proc 4th Int Conf Physiological Fluid dynamics, Gwalior pp - - Dec 8-10, (1995)